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Zeronic Field and its Intrinsic Core

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Abstract

Zeron is the most elementary particle which is the quantum of essential quantization in proper mass system (proper system for short) - when its speed is less than c it is an implicit particle; but when its speed is equal to c it becomes an explicit particle. An infinite set of zerons distributed in whole space all over the entire space-time can form a field - zeronic field, of which either a zero-proper-mass particle or a nonzero-proper-mass particle can be composed. There exists a stable intrinsic core in a zeronic field, from which many important fundamental properties in particle physics such as the electron mass can be calculated, when that the de Broglie wave corresponds to not only a relativistic frequency but also a relativistic waving vector is reasoned. This document is written using only two parameters $\hbar$ and c.

Key words: essential quantization, zeron, zeronic field with intrinsic core

The quantization of proper or intrinsic mass is an essential quantization, which can be derived directly from Einstein field equation as well as whether from the ϕ^4 theory in quantum field theory [Ref. 1] or from the special relativistic mechanics [Ref. 2], even from pure mathematics when differential geometry was initially established on curves and surfaces.

1 Essential quantization derived from helix sequence

Helix is a typical curve since differential geometry was originally established on curves and surfaces about 200 years ago. A 3-dimension helix wave, formed as a point moving along the helix, also represents an essential waving form and can be transferred to 2-dimension waving motion even 1-dimension harmonic motion. As a curve, helix has curvature and torsion, those are very important variables because their extreme values play key roles in quantization.

1.1 *Helix curve and helix sequence*

The parameterized differentiable curve given by [Ref 5: Chap.3, Sect.1]

$$\begin{aligned} \boldsymbol{r} &= \{a\cos\varphi, a\sin\varphi, b\varphi\} \quad (a>0, \varphi\in R, b\neq 0) \\ &= a\cos\varphi\boldsymbol{e}_1 + a\sin\varphi\boldsymbol{e}_2 + b\varphi\boldsymbol{e}_3 \end{aligned} \tag{1}$$

has as its trace in R^3 a helix of pitch $2\pi b$ on the cylinder $x^2+y^2=a^2$. The parameter φ here means the angle that the x axis makes with the line joining the origin 0 to the

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projection of the point $\boldsymbol{r}(\varphi)$ over the $x-y$ plane. The $\boldsymbol{e}$'s here are orthogonal base vectors.

The helix curvature is

$$k=\frac{a}{a^2+b^2} \tag{2}$$

and the helix torsion is

$$\tau=\frac{b}{a^2+b^2} \tag{3}$$

where $b<0$ for left-revolving helix while $b>0$ for right-revolving helix. To get the extreme value of the curvature k as varying with a, must

$$a^2=b^2. \tag{4}$$

This time, the torsion τ varying with b also gets the extreme values.

Differentiating $\boldsymbol{r}$,

$$\mathrm{d}\boldsymbol{r}=\{-a\sin\varphi, a\cos\varphi, b\}\,\mathrm{d}\varphi. \tag{5}$$

Hence

$$(\mathrm{d}\boldsymbol{r})^2=(\mathrm{d}s)^2=\left(a^2+b^2\right)(\mathrm{d}\varphi)^2 \quad (s\text{ is arc length}). \tag{6}$$

If the increasing trend of s is the same as φ,

$$\mathrm{d}s=\sqrt{a^2+b^2}\,\mathrm{d}\varphi=R_0\mathrm{d}\varphi \overset{a^2=b^2}{===} \sqrt{2}a\mathrm{d}\varphi \quad \left(R_0=\sqrt{a^2+b^2}\overset{a^2=b^2}{===}\sqrt{2}a\right) \tag{7}$$

where R_0 is the radial vector. Now, as a basic vestor, unit tangent vector is

$$\boldsymbol{\alpha}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s}=\dot{\boldsymbol{r}}=\frac{\text{expression (5)}}{\text{expression (7)}}=\frac{1}{\sqrt{a^2+b^2}}\{-a\sin\varphi, a\cos\varphi, b\}. \tag{8}$$

Then

$$\dot{\boldsymbol{\alpha}}=\frac{\mathrm{d}\boldsymbol{\alpha}}{\mathrm{d}s}=\ddot{\boldsymbol{r}}=\frac{a}{a^2+b^2}\{-\cos\varphi, -\sin\varphi, 0\} \overset{a^2=b^2}{===} \frac{1}{\sqrt{2}R_0}\{-\cos\varphi, -\sin\varphi, 0\}. \tag{9}$$

According to Frenet-Serret formulae, $\dot{\boldsymbol{\alpha}}=k\boldsymbol{\beta}$, where $\boldsymbol{\beta}$ is another basic vector named (unit) normal vector. Thus,

$$k=\frac{a}{a^2+b^2}=\frac{1}{R}\overset{a^2=b^2}{===}\frac{1}{2a}=\frac{1}{\sqrt{2}R_0}, \tag{10}$$

$$\boldsymbol{\beta}=\{-\cos\varphi, -\sin\varphi, 0\}. \tag{11}$$

The

$$R=\frac{a^2+b^2}{a}\overset{a^2=b^2}{===}2a=\sqrt{2}R_0 \tag{12}$$

is the radius of curvature. Furthermore,

$$\dot{\boldsymbol{\alpha}}^2=\ddot{\boldsymbol{r}}^2=k^2=\frac{1}{R^2}. \tag{13}$$

When $b=|b|>0$,

$$\boldsymbol{r}=\{a\cos\varphi, a\sin\varphi, |b|\varphi\}=a\cos S\boldsymbol{e}_1+a\sin S\boldsymbol{e}_2+|b|S\boldsymbol{e}_S \quad \left(S=\varphi=\frac{s}{R_0},\ \boldsymbol{e}_S=\boldsymbol{e}_3\right) \quad (14)$$

is right-revolving helix. If compared with the expression of helicoid [Ref. 5: Chap. 5, Sect. 4 and Chap. 6 Sect. 2]

$$\boldsymbol{r}=\{u\cos v, u\sin v, bv\} \quad (-\infty<v<\infty) \quad (15)$$

where v is helix too, the helix expression (1)

$$\boldsymbol{r}=\{a\cos\varphi, a\sin\varphi, b\varphi\} \quad (-\infty<\varphi<\infty) \quad (16)$$

has the exactly same written form and, of course, the φ in (1) and (16) or the $S=\varphi$ in (14) also represents another helix. Thus rewriting above expression (14) as

$$-|b|S\boldsymbol{e}_S=a\cos S\boldsymbol{e}_1+a\sin S\boldsymbol{e}_2-|\boldsymbol{r}|\boldsymbol{e}_r \quad (|\boldsymbol{r}|\boldsymbol{e}_r=\boldsymbol{r}), \quad (17)$$

it has been a new helix. To compare the $-|\boldsymbol{r}|$ before $\boldsymbol{e}_r$ in expression（17）with the $+|b|$ before $\boldsymbol{e}_S$ in expression（14） proves the new helix has turned into left-revolving one. Meanwhile, that the $+|b|S\boldsymbol{e}_S$ is changed to $-|b|S\boldsymbol{e}_S$ proves the increasing directions of new and old helices are opposite. In order to distinguish these two different helices, writing the new helix into

$$\boldsymbol{r}^{(1)}=-|b|S\boldsymbol{e}_s^{(1)}=a\cos S\boldsymbol{e}_1^{(1)}+a\sin S\boldsymbol{e}_2^{(1)}-|\boldsymbol{r}|\boldsymbol{e}_{r^{(0)}} \quad \left(\boldsymbol{e}_{r^{(0)}}=\boldsymbol{e}_r\right), \quad (18)$$

then the old helix may be expressed as

$$\boldsymbol{r}^{(0)}=a\cos S\boldsymbol{e}_1^{(0)}+a\sin S\boldsymbol{e}_2^{(0)}+|b|S\boldsymbol{e}_S \quad \left(\boldsymbol{e}_n^{(0)}=\boldsymbol{e}_n,\ n=1,2,3\right). \quad (19)$$

Note that the new helix with both revolving and increasing directions oppositely changed is actually a helix taking the old helix as its revolving axis. All helices formed through such a manner one by one can be summarized as

$$\boldsymbol{r}^{(i)}=a\cos S\boldsymbol{e}_1^{(i)}+a\sin S\boldsymbol{e}_2^{(i)}(-)^i|b|S\boldsymbol{e}_3^{(i)}$$
$$\left(i=1,2,3,\cdots,\ |b|S=\left|\boldsymbol{r}^{(i-1)}\right|,\ a^2=b^2,\ S>>1,\ \boldsymbol{e}_3^{(i)}=\boldsymbol{e}_{r^{(i-1)}}\right), \quad (20)$$

where

$$\boldsymbol{e}^{(i)}\not\equiv\boldsymbol{e}^{(i-1)}. \quad (21)$$

Every $\boldsymbol{r}^{(i+1)}$ curve is a helix rotating around $\boldsymbol{r}^{(i)}$ as its revolving axis, there are infinite multiplicity of such helices to constitute an infinite helix sequence in proper space. Then an important fact is that $\boldsymbol{r}^{(i+1)}$ has not only been opposite in revolving and increasing directions but also possessed secondary structure [Ref. DNA structure in biochemistry] which is right the $\boldsymbol{r}^{(i)}$ structure.

1.2 *Moving speed along a curve must have a limit*

The relation among the curvature k, the normal curvature k_n and the geodesic curvature k_g of a curve on a surface is [Ref. 5: Chap. 8, Sect. 5. expression (6)]

$$k^2=k_n^2+k_g^2 \quad (22)$$

where

$$k_n=k\cos\theta \quad \text{and} \quad |k_g|=k\sin\theta, \quad (23)$$

θ is magnitude of the angle between the normal vector $\boldsymbol{\beta}$ of the curve and the normal vector $\boldsymbol{n}$ of the surface at the same point. Thus

$$k^2 = \left(k\cos\theta\right)^2 + \left(k\sin\theta\right)^2. \tag{24}$$

When $k \neq 0$,

$$\mathbf{e} = \cos\theta\mathbf{e}_i + \sin\theta\mathbf{e}_j \quad \mathbf{e}_i \cdot \mathbf{e}_j = 0 \quad |\mathbf{e}_i| = |\mathbf{e}_j| = |\mathbf{e}| = 1 \tag{25}$$

where $\mathbf{e}$ indicates any one of the three unit vectors in Frenet frame in which the unit tangent vector

$$\boldsymbol{\alpha} = \frac{\boldsymbol{u}}{|u|} \tag{26}$$

is unit velocity. As either $\alpha\cos\theta$ or $\alpha\sin\theta$ is less than or equals to the unit 1, so the α corresponds to the maximal speed - light speed c - as a limit. The above vector equation then becomes

$$c\boldsymbol{\alpha} = c\cos\theta\boldsymbol{\alpha}_i + c\sin\theta\boldsymbol{\alpha}_j = |u|\boldsymbol{\alpha}_i + \sqrt{c^2 - u^2}\boldsymbol{\alpha}_j \quad \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j = 0 \quad |\boldsymbol{\alpha}_i| = |\boldsymbol{\alpha}_j| = |\boldsymbol{\alpha}| = \alpha \tag{27}$$

when choosing $\cos\theta = |u|/c$ and $\sin\theta = \sqrt{c^2 - u^2}/c$. Because what is analysed now is a curve with its $k \neq 0$ on a surface and a point moving along this curve must have its angular velocity $\boldsymbol{\omega}$ whose value $|\boldsymbol{\omega}| = c/R$ is angular frequency [Ref. 4: pp. 32-33 plus p.515], R is the radius of curvature. Thus

$$|\boldsymbol{\omega}|\boldsymbol{\alpha} = |\boldsymbol{\omega}|\left(|u|/c\right)\boldsymbol{\alpha}_i + |\boldsymbol{\omega}|\sqrt{1 - u^2/c^2}\boldsymbol{\alpha}_j \qquad \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j = 0 \tag{28}$$

where

$$\boldsymbol{\omega} = |\boldsymbol{\omega}|\boldsymbol{\alpha} = |\omega|\boldsymbol{\alpha} \tag{29}$$

is total waving velocity,

$$|\boldsymbol{\omega}|(|u|/c)\boldsymbol{\alpha}_i = |\boldsymbol{\omega}|\cos\theta\boldsymbol{\alpha}_i = \boldsymbol{\omega}_{\mathrm{v}} \tag{30}$$

is kinetic waving velocity while

$$|\boldsymbol{\omega}|\sqrt{1 - u^2/c^2}\boldsymbol{\alpha}_j = |\boldsymbol{\omega}|\sin\theta\boldsymbol{\alpha}_j = \boldsymbol{\omega}_0 \tag{31}$$

is proper waving velocity, and

$$\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathrm{v}} + \boldsymbol{\omega}_0 \quad \boldsymbol{\omega}_{\mathrm{v}} \cdot \boldsymbol{\omega}_0 = 0 \qquad \omega^2 = \omega_{\mathrm{v}}^2 + \omega_0^2. \tag{32}$$

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1.3 *Discreteness of helix sequence*

An important equality should be proved now, it is

$$R^{(n+i)} = \left(\sqrt{2}\right)^i R^{(n)} \quad \left(a^2 = b^2,\ n, i = 0, 1, 2, \cdots\right). \tag{33}$$

Taking $i = 1$, when $n = 0$ （33）is right in expression（12）only need to assign $R^{(0)} = R_0$ and $R^{(1)} = R$. If when $n = j$, $R^{(j+1)} = \left(\sqrt{2}\right)R^{(j)}$ (still taking $i = 1$) is correct, then when $n = j + 1$

$$R^{(j+2)} = \sqrt{2}R^{(j+1)} = \sqrt{2}\sqrt{2}R^{(j)} = \left(\sqrt{2}\right)^2 R^{(j)}, \tag{34}$$

meeting （33）, so the equality （33）can be set up when $i=1$. In general, if $i=k\ (k=0,1,2,\cdots)$ still （33）is an available one as

$$R^{(n+k)}=\left(\sqrt{2}\right)^{k}R^{(n)}, \tag{35}$$

then when $i=k+1$,

$$R^{[n+(k+1)]}=R^{[(n+k)+1]}=\sqrt{2}R^{(n+k)}=\sqrt{2}\left(\sqrt{2}\right)^{k}R^{(n)}=\left(\sqrt{2}\right)^{k+1}R^{(n)}. \tag{36}$$

Thus equality （33）is set up whatever natural-number i takes.

According to expression （33）, there exists

$$k^{(i)}=\frac{1}{R^{(i)}}=\frac{1}{\left(\sqrt{2}\right)^{1}R^{(i-1)}}=\frac{1}{\left(\sqrt{2}\right)^{2}R^{(i-2)}}=\cdots=\frac{1}{\left(\sqrt{2}\right)^{i}R^{(i-i)}}=\frac{1}{\left(\sqrt{2}\right)^{i}R^{(0)}}\qquad (i=1,2,3,\cdots), \tag{37}$$

equivalent to （33）when $n=0$, i.e.,

$$R^{(i)}=\left(\sqrt{2}\right)^{i}R^{(0)}=\left(\sqrt{2}\right)^{i}R_{0}\qquad (\ i=0,1,2,\cdots). \tag{38}$$

When a point moves at light-speed c on a curve with curvature $k=1/R\neq 0$ and its angular frequency $|\boldsymbol{\omega}|=c/R$,

$$R^{2}=\left(\frac{c}{\omega}\right)^{2}. \tag{39}$$

If moving on each $\boldsymbol{r}^{(i)}\ (i=1,2,3,\cdots)$,

$$\left(\ddot{\boldsymbol{r}}^{(i)}\right)^{2}=\left(k^{(i)}\right)^{2}=\frac{1}{\left(R^{(i)}\right)^{2}}=\left(\frac{\omega^{(i)}}{c}\right)^{2}\quad (\omega^{(0)}=\omega). \tag{40}$$

That is

$$\left(\omega^{(i)}\right)^{2}=\left(\frac{c}{R^{(i)}}\right)^{2}=\left(\frac{c}{\left(\sqrt{2}\right)^{i}R_{0}}\right)^{2}=\left(\frac{\omega_{0}}{\left(\sqrt{2}\right)^{i}}\right)^{2}\qquad \left(i=1,2,3,\cdots,|\omega_{0}|=\frac{c}{R_{0}}\right), \tag{41}$$

which means, if $\omega_{0}^{(i)}$ is marked to replace $\omega^{(i)}$ here,

$$\left|\omega_{0}^{(i)}\right|=\frac{|\omega_{0}|}{\left(\sqrt{2}\right)^{i}}\qquad (i=1,2,3,\cdots). \tag{42}$$

This is describing discreteness of helix sequence.

1.4 *Essential quantization derived from helix sequence*

The frequency $|\boldsymbol{\omega}|$ is waving frequency, together with the wave amplitude a and the wave phase φ, the parameterized differentiable expressions of helix represent not merely the helical curve but also an essential sort of wave and waving form for all in motion including matter.

As matter wave, de Broglie wave [Ref. 4: p. 826] results in the relation about wave-particle duality

$$mc^2 = \hbar|\omega| \ \ (m = m^{(0)}) \ \Rightarrow \ m_0 c^2 = \hbar|\omega_0| \tag{43}$$

where $|\omega|$ is de Broglie wave angular frequency corresponding to particle mass m while m_0 is rest mass to which $|\omega_0|$ is corresponding and $\hbar$ is Planck constant (reduced).

Let

$$\left(\omega^{(i)}\right)^2 - \left(\omega_0^{(i)}\right)^2 = \left(\omega_{\rm v}^{(i)}\right)^2 \quad (i = 0,1,2,\cdots,\ \omega^{(0)} = \omega,\ \omega_{0/\rm v}^{(0)} = \omega_{0/\rm v},\ \omega^{(i)} = \omega_0^{(i-1)} \text{ for } i > 0) \tag{44}$$

meeting

$$\left(\omega_0^{(i)}\right)^2 = \left(\omega_{\rm v}^{(i)}\right)^2 = \frac{\left(\omega_0^{(i-1)}\right)^2}{2} \quad (i = 1,2,3,\cdots), \tag{45}$$

where $\left(\omega^{(i)}\right)^2$ is the sum of $\left(\omega_0^{(i)}\right)^2$ on time dimension plus $\left(\omega_{\rm v}^{(i)}\right)^2$ on moving direction dimension [Ref. 3：expressions (40) to (52)] while $\left(\omega_0^{(i)}\right)^2$ will also have four dimensions when being the sum of $\left(\omega_0^{(i+1)}\right)^2$ and $\left(\omega_{\rm v}^{(i+1)}\right)^2$. Then

$$\boldsymbol{\omega}^{(i)} - \boldsymbol{\omega}_0^{(i)} = \boldsymbol{\omega}_{\rm v}^{(i)} \quad \boldsymbol{\omega}_0^{(i)} \cdot \boldsymbol{\omega}_{\rm v}^{(i)} = 0 \quad (i = 0,1,2,\cdots,\ \boldsymbol{\omega}^{(0)} = \boldsymbol{\omega},\ \boldsymbol{\omega}_{0/\rm v}^{(0)} = \boldsymbol{\omega}_{0/\rm v},\ \boldsymbol{\omega}^{(i)} = \boldsymbol{\omega}_0^{(i-1)} \text{ for } i > 0). \tag{46}$$

So there is vector wave [Ref. 4: p. 288, p. 292 plus p. 817] that belongs to electromagnetic wave formed in light-speed motion.

Via expressions (42) and (43), the m_0 becomes discrete rest mass

$$m_0^{(i)} = \frac{m_0}{\left(\sqrt{2}\right)^i} \quad (\ i = 1,2,3,\cdots;\ m_0 = m_0^{(0)}) \tag{47}$$

together with

$$m_0^{(i)} c^2 = \hbar\left|\omega_0^{(i)}\right| \quad (i = 0,1,2,\cdots;\ m_0^{(0)} = m_0) \tag{48}$$

and

$$m_{\rm v}^{(i)} c^2 = \hbar\left|\omega_{\rm v}^{(i)}\right| \quad (i = 0,1,2,\cdots;\ m_{\rm v}^{(0)} = m_{\rm v}). \tag{49}$$

Since

$$mr = \frac{\hbar|\omega|}{c^2} r = \frac{\hbar}{c} \quad \text{when } |\omega| r = c, \tag{50}$$

if m or $|\omega|$ is big, r and wave length must be small. Only during m is being small enough to be near m_0 in microphysics, the wave length becomes large enough to be detected and the intrinsic helix wave with this microeffect will be manifest in the locality.

Relying on expressions (42) and (47) under such microeffect, helix sequence system has been quantized proper mass system (proper system for short) in which expressions (42) and (47) have been the quantization about either mass m_0 of $m_0 c^2$ or frequency $|\omega_0|$ of $\hbar|\omega_0|$ and called essential quantization in proper system.

Now, 1-dimension $\boldsymbol{\omega}_0^{(0)}$ plus 3-dimension helix wave $\boldsymbol{\omega}_{\rm v}^{(0)}$ constitute $\boldsymbol{\omega}$ in complete 4-dimension space-time while $\boldsymbol{\omega}_0^{(k)}$ $(k \geq 1)$ plus $\boldsymbol{\omega}_{\rm v}^{(k)}$ $(k \geq 1)$ constitute $\boldsymbol{\omega}_0^{(k-1)}$ in proper 4-dimension space-time in which there is existing a proper system with essential quantization; and mc^2 is particle energy of the light-speed moving point along the helix curve while equalled $\hbar|\omega|$ is waving energy of the helix wave.

As known [Ref. Subsection 1.2] $\boldsymbol{\omega}_{\rm v} = \boldsymbol{\omega}_{\rm v}^{(0)}$ is called kinetic waving velocity, while present $m_{\rm v} = m_{\rm v}^{(0)}$ is called kinetic (proper) mass. According to above expressions, let

$$\left|\omega_0^{(i)}\right| - \left|\omega_0^{(i+1)}\right| = \frac{\left(\omega_{\rm v}^{(i+1)}\right)^2}{\left|\omega_0^{(i)}\right| + \left|\omega_0^{(i+1)}\right|} = \left|\omega_{\rm f}^{(i+1)}\right| \quad (i = 0,1,2,\cdots)\,, \tag{51}$$

then

$$\omega_{\rm f}^{(i+1)} = \omega_0^{(i)} \mp \left|\omega_0^{(i+1)}\right| \begin{cases} > 0 \ \text{ if } \omega_0^{(i)} > 0 \\ < 0 \ \text{ if } \omega_0^{(i)} < 0 \end{cases} \quad (i = 0,1,2,\cdots) \tag{52}$$

when $\left|\omega_{\rm f}\right| = \left|\omega_{\rm f}^{(0)}\right| = |\omega| - |\omega_0|$ is called dynamic waving frequency, that is

$$m_0^{(i)} - m_0^{(i+1)} = \frac{\left(m_{\rm v}^{(i+1)}\right)^2}{m_0^{(i)} + m_0^{(i+1)}} = m_{\rm f}^{(i+1)} \quad (i = 0,1,2,\cdots) \tag{53}$$

when $m_{\rm f} = m_{\rm f}^{(0)} = m - m_0$ is called dynamic (proper) mass.

Helix is the geodesic line on cylindrical surface [Ref 5: Chap.8, Subsection 6.1 (iii)]. The geodesic line keeps geodesic curvature $\left|k_{\rm g}\right| = k\sin\theta \equiv 0$ [Ref 5: Cha p.8, Subsection 6.1], that is, $\cos\theta = \frac{|u|}{c} \equiv 1 \Rightarrow |u| \equiv c$ [Ref. Subsection 1.2]. So, helix expression (1) is the free-moving path for any zero-proper-mass particle.

2 Zeron

As

$$\boldsymbol{\omega}_{\rm v}^{(i)} = c\boldsymbol{\kappa}^{(i)} = c\boldsymbol{p}^{(i)}/\hbar = cm_0^{(i-1)}\boldsymbol{u}^{(i)}/\hbar = (m_0^{(i-1)}c^2)\boldsymbol{u}^{(i)}/\hbar c = \frac{(\hbar\,|\,\omega_0^{(i-1)}\,|)\boldsymbol{u}^{(i)}}{\hbar c} = \frac{|\,\omega_0\,|\,\boldsymbol{u}^{(i)}}{\left(\sqrt{2}\right)^{i-1} c} \tag{54}$$

$$(i = 1,2,3,\cdots)$$

where $\boldsymbol{\kappa}$ is wave vector and $\boldsymbol{p} = m\boldsymbol{u}$ is momentum, and

$$\boldsymbol{\omega}_{\rm v} = c\boldsymbol{\kappa} = c\boldsymbol{p}/\hbar = cm\boldsymbol{u}/\hbar = (mc^2)\boldsymbol{u}/\hbar c = \frac{(\hbar\,|\,\omega\,|)\boldsymbol{u}}{\hbar c} = \frac{|\,\omega\,|\,\boldsymbol{u}}{c} \quad (\boldsymbol{u} = \boldsymbol{u}^{(0)}) \tag{55}$$

corresponding

$$m_{\rm v}^{(i)} = \frac{\left|p^{(i)}\right|}{c} = \frac{m_0^{(i-1)}\left|u^{(i)}\right|}{c} = \frac{m_0}{\left(\sqrt{2}\right)^{i\text{-}1}}\frac{\left|u^{(i)}\right|}{c} \quad (i = 1,2,3,\cdots)\,, \tag{56}$$

and

$$m_{\mathrm{v}}=\frac{|p|}{c}=\frac{m|u|}{c}. \tag{57}$$

Note that

$$\begin{aligned}\boldsymbol{\omega}=\boldsymbol{\omega}_{\mathrm{v}}+\boldsymbol{\omega}_0=\boldsymbol{\omega}_{\mathrm{v}}+\boldsymbol{\omega}_{\mathrm{v}}^{(1)}+\boldsymbol{\omega}_0^{(1)}=\cdots=\boldsymbol{\omega}_{\mathrm{v}}+\sum_{i=1}^{\infty}\boldsymbol{\omega}_{\mathrm{v}}^{(i)}=\frac{|\omega|}{c}\boldsymbol{u}^{(0)}+\left[\frac{|\omega_0|}{c}\sum_{i=1}^{\infty}\frac{\boldsymbol{u}^{(i)}}{(\sqrt{2})^{i-1}}\right]_{u=0}\\ \Rightarrow\ c\mathbf{e}=c\frac{\boldsymbol{\omega}}{|\omega|}=\boldsymbol{u}+\left[\frac{|\omega_0|}{|\omega|}\sum_{i=1}^{\infty}\frac{\boldsymbol{u}^{(i)}}{(\sqrt{2})^{i-1}}\right]_{u=0}\quad\left(\mathbf{e}=\frac{\boldsymbol{\omega}}{|\omega|}\right)\end{aligned} \tag{58}$$

where

$$\boldsymbol{\omega}_0=\sum_{i=1}^{\infty}\boldsymbol{\omega}_{\mathrm{v}}^{(i)}\equiv\left[\frac{|\omega_0|}{c}\sum_{i=1}^{\infty}\frac{\boldsymbol{u}^{(i)}}{(\sqrt{2})^{i-1}}\right]_{u=0} \tag{59}$$

is proper quantity.

Now, whether $m_0=0$, that is, $\boldsymbol{\omega}_0=0$ and $\boldsymbol{\omega}=\boldsymbol{\omega}_{\mathrm{v}}\ \Rightarrow$

$$c\mathbf{e}_{\mathrm{v}}=c\frac{\boldsymbol{\omega}_{\mathrm{v}}}{|\omega_{\mathrm{v}}|}=\boldsymbol{u}\quad\left(\mathbf{e}_{\mathrm{v}}=\frac{\boldsymbol{\omega}_{\mathrm{v}}}{|\omega_{\mathrm{v}}|}\right); \tag{60}$$

or $\boldsymbol{u}=0$, that is, $|\omega|=|\omega_0|$ and $\boldsymbol{\omega}=\boldsymbol{\omega}_0\ \Rightarrow$

$$c\mathbf{e}_0=c\frac{\boldsymbol{\omega}_0}{|\omega_0|}=\sum_{i=1}^{\infty}\frac{\boldsymbol{u}^{(i)}}{(\sqrt{2})^{i-1}}\quad\left(\mathbf{e}_0=\frac{\boldsymbol{\omega}_0}{|\omega_0|}\right). \tag{61}$$

Therefore,

$$c\mathbf{e}=\boldsymbol{u}+c\mathbf{e}_0, \tag{62}$$

this is vector expression about constant light-speed, while light-ray may be refracted. Only when the unit vectors $\mathbf{e}$, $\mathbf{e}_0$ and $\mathbf{e}_{\mathrm{v}}$ have been fixed meeting this expression, any length, such as $|\boldsymbol{u}|$ or $|\boldsymbol{\omega}_{(0/\mathrm{v})}|$, can be determined.

Above conclusions have no any contradiction, because both massless and massive particles are already united as zeron − zero-proper-mass particle.

What is resting when $m=m_0^{(i)}$ is only a center-of-rotation (or say center-of-curvature) of intrinsic $\boldsymbol{\omega}_0^{(i)}$. Such a point is a zeron. Its characteristic can be shown as

$$Q=\left.\frac{Q_0}{\sqrt{1-u^2/c^2}}\right|_{Q_0=0}=\begin{cases}0 & \text{when } u^2\neq c^2\\ \text{some constant} & \text{when } u^2=c^2\end{cases} \tag{63}$$

where Q may be $m,\omega,p\left(=|\boldsymbol{p}|\right),\kappa,E$ (energy) and other relevant quantities, Q_0 takes the value of Q when $m_0=0$ and $u=0$. When the zeron's speed is less than c, it is an implicit particle; but when its speed is equal to c, it becomes an explicit zero-proper-mass particle.

Let

$$u^{(i)}=\pm c\sqrt{1-\left(\frac{m_0^{(j)}}{m_0^{(i)}}\right)^2}\qquad(i>0). \tag{64}$$

Then

$$u^{(i)} = \begin{cases} 0 \ \text{when}\ j = i \\ \pm\dfrac{c}{\sqrt{2}} \ \text{when}\ j = i+1 \\ \pm c \ \text{when}\ j = \infty \\ \pm \mathrm{i}c \ \text{when}\ j = i-1 \end{cases}. \tag{65}$$

* (This paragraph is as additional reference) The proof of wave-particle duality can be also given for a Dirac particle in quantum field theory - *Dirac particle is also a massless quantum*.

The Lagrangian for a spinor (spin - $\frac{1}{2}$) field is

$$\mathcal{L} = i(\hbar c)\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \left(m_0 c^2\right)\bar{\psi}\psi \tag{66}$$

where m_0 is the proper mass, either ψ or $\bar{\psi}$ is an independent field variable. Applying the Euler-Lagrange equation to ψ, the adjoint Dirac equation

$$i\partial_{\mu}\bar{\psi}\gamma^{\mu} + \left(\frac{m_0 c}{\hbar}\right)\bar{\psi} = 0 \tag{67}$$

can be established adjoining the Dirac equation

$$i\gamma^{\mu}\partial_{\mu}\psi - \left(\frac{m_0 c}{\hbar}\right)\psi = 0. \tag{68}$$

Then the Lagrangian above becomes

$$\begin{aligned} \mathcal{L} &= i(\hbar c)\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \left[-i(\hbar c)\partial_{\mu}\bar{\psi}\gamma^{\mu}\right]\psi \\ &= i(\hbar c)\left[\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] \\ &= i(\hbar c)\partial_{\mu}\left(\bar{\psi}\gamma^{\mu}\psi\right) \equiv 0 \end{aligned} \tag{69}$$

because the vector current $\bar{\psi}\gamma^{\mu}\psi$ is always conserved if ψ satisfies the Dirac equation. Thus the Hamiltonian is

$$H = \text{constant}. \tag{70}$$

That there is no mass term here in the Lagrangian and there is no varying of energy accumulated from the whole space proves that a definite Dirac field represents not only a massive Dirac particle but also a massless field quantum. As known, any massless, i.e., zero-proper-mass quantum is a zeron, both a zero-proper-mass particle such as a photon and a nonzero-proper-mass particle such as an electron are zerons.

3 Zeronic field and its intrinsic core

3.1 *Zeronic field*

Let, if $\boldsymbol{r}$ is the radius of curvature,

$$c\boldsymbol{\kappa}^0 = c\boldsymbol{\varpi}^0 = \boldsymbol{\varpi}\times\boldsymbol{r} \quad \left(\boldsymbol{\varpi}\cdot\boldsymbol{r} = 0, \boldsymbol{\kappa}^0 = \frac{\boldsymbol{\kappa}}{\kappa}, \boldsymbol{\varpi}^0 = \frac{\boldsymbol{\varpi}}{\varpi},\ r > 0\right) \tag{71}$$

and

$$c = |\omega| r \qquad \omega r = \pm c\ , \tag{72}$$

or,

$$\begin{gathered} c\left(\boldsymbol{\omega}_{0/\mathrm{v}}^{(i)}\right)^{0} = \left(\boldsymbol{\omega}\right)_{0/\mathrm{v}}^{(i)} \times \boldsymbol{r}_{0/\mathrm{v}}^{(i)} \\ \left[\left(\boldsymbol{\omega}\right)_{0/\mathrm{v}}^{(i)} \cdot \boldsymbol{r}_{0/\mathrm{v}}^{(i)} = 0, \left(\boldsymbol{\omega}_{0/\mathrm{v}}^{(i)}\right)^{0} = \boldsymbol{\omega}_{0/\mathrm{v}}^{(i)} \big/ \omega_{0/\mathrm{v}}^{(i)},\ r_{0/\mathrm{v}}^{(i)} > 0, r_{0/\mathrm{v}}^{(0)} = r_{0/\mathrm{v}}\right] \end{gathered} \tag{73}$$

and

$$c = \left|\omega_{0/\mathrm{v}}^{(i)}\right| r_{0/\mathrm{v}}^{(i)} \qquad \omega_{0/\mathrm{v}}^{(i)} r_{0/\mathrm{v}}^{(i)} = \pm c, \tag{74}$$

where

$$r_{\mathrm{v}}^{(i+1)} = r_{0}^{(i+1)} \quad (i > 0)\,. \tag{75}$$

Name $\boldsymbol{\omega}$ total (waving) angular velocity, $\boldsymbol{\omega}_{\mathrm{v}}$ kinetic (waving) angular velocity and $\boldsymbol{\omega}_0$ proper (waving) angular velocity. When $\omega_{0/\mathrm{v}}^{(i)}$ or $m_{0/\mathrm{v}}^{(i)} \to 0$,

$$r_{0/\mathrm{v}}^{(i)} \xrightarrow{\omega_{0/\mathrm{v}}^{(i)} \text{ or } m_{0/\mathrm{v}}^{(i)} \xrightarrow{i\to\infty} 0} \infty. \tag{76}$$

This proves that with the infinite multiplicity of zerons, a nonzero-proper-mass particle is composed of

$$m_0 = \frac{\hbar\left|\boldsymbol{\omega}_0\right|}{c^2} \tag{77}$$

while

$$\boldsymbol{\omega}_0 = \sum_{i=1}^{\infty} \boldsymbol{\omega}_{\mathrm{v}}^{(i)} \tag{78}$$

with

$$c\frac{\boldsymbol{\omega}_{\mathrm{v}}^{(i)}}{\left|\omega_{\mathrm{v}}^{(i)}\right|} = \sqrt{2}\boldsymbol{u}^{(i)} \quad \left(i = 1, 2, 3, \cdots, \left|\omega_{\mathrm{v}}^{(i)}\right| = \left|\omega_{0}^{(i)}\right|\right) \tag{79}$$

that means

$$\boldsymbol{u}^{(i)} = \frac{c}{\sqrt{2}}\frac{\boldsymbol{\omega}_{\mathrm{v}}^{(i)}}{\left|\omega_{\mathrm{v}}^{(i)}\right|} = \frac{c}{\sqrt{2}}\mathbf{e}_{\mathrm{v}}^{(i)} \quad \left(i = 1, 2, 3, \cdots, \left|\omega_{\mathrm{v}}^{(i)}\right| = \left|\omega_{0}^{(i)}\right|,\ \ \mathbf{e}_{\mathrm{v}}^{(i)} = \frac{\boldsymbol{\omega}_{\mathrm{v}}^{(i)}}{\left|\omega_{\mathrm{v}}^{(i)}\right|}\right). \tag{80}$$

Plus expression (58), this waving particle of infinite wave length is distributed in whole space all over the entire space-time to form a field − zeronic field. It is the relative effect

$$0 < 2\pi r'' = 2\pi r' \sqrt{1 - \left(\frac{u}{c}\right)^2}\Bigg|_{r'=\infty, u=c} < \infty \tag{81}$$

that makes this particle shown as a revolving zeron at the speed of c within a finite space.

The definite center of any intrinsic rotation is also the center-of-mass of m_{t} - the total mass in the universe except the mass m itself, and this mass m is actually the reduced mass of m_{t} and itself for that

$$\frac{m_{\mathrm{t}} m}{m_{\mathrm{t}} + m} \xrightarrow{m_{\mathrm{t}} \to \infty} m. \tag{82}$$

P.S. Zeronic field is right ether field

1. *Without external impact no light speed will be changed*

Expression (62) proves that light speed will not be changed in zeronic field. So, the light speed is not an issue for zeronic field to be ether field.

2. *Zeron is both implicit medium and explicit field component*

When $u^2 \neq c^2$, expression (63) shows the zeron is implicit medium without any impact on others in zeronic field, such as permittivity $\varepsilon = n^2$ [Ref, 6: p. 258], n is refractive index. In Snell's law of refraction [Ref 4: p. 346] here

$$\frac{\sin\theta_{\mathrm{i}}}{\sin\theta_{\mathrm{r}}} = \frac{n_{\mathrm{r}}}{n_{\mathrm{i}} \equiv \mathrm{Const.}} = \frac{v_{\mathrm{i}} = |c\mathbf{e}_0|}{v_{\mathrm{r}} = |c\mathbf{e}|} \tag{83}$$

where θ_{i} and θ_{r} are angles of incidence and refraction respectively, n_{i} and n_{r} are refractive indices of incidence side and refraction side respectively while v_{i} and v_{r} are respective absolute phase velocity of incidence side and refraction side. So the permittivity of refraction side $\varepsilon_{\mathrm{r}} = n_{\mathrm{r}}^2$ in zeronic field keeps constant.

3. *Action at a distance is actually "action within R_0"*

Expression (81) proves zeronic field is both whole space waving field and particulate region of local space, this is why it displays wavicle at the same time. If rewrite (81) as another contracting expression,

$$R^{(j)} = R_0 \prod_{i=1}^{j} \left(\sqrt{2}\right)^i \sqrt{1-\left(\frac{u^{(i)}}{c}\right)^2}\Bigg|_{u^{(i)}=\pm\frac{c}{\sqrt{2}}} = R_0 \prod_{i=1}^{j}\left[\left(\sqrt{2}\right)^i \Big/ \left(\sqrt{2}\right)^i\right] \equiv R_0 \tag{84}$$

even $j \to \infty$, hence when $R^{(j)}$ becomes $R^{(j+1)}$, the two helices may interact at distance $a \overset{a^2=b^2}{===} \frac{\sqrt{a^2+b^2}}{\sqrt{2}} = \frac{R_0}{\sqrt{2}}$ within R_0, as mentioned after expression (21), and the so-called "Action at a distance" may actually be "action within R_0" though there could have a long long even infinite long travel along the helices.

3.2 *Intrinsic core – intrinsic construction of zeronic field*

Although all $\boldsymbol{\omega}_{\mathrm{v}}^{(i+1)}$'s $(i \geq 0)$ are completely intrinsic quantities, yet their vectorial sum $\boldsymbol{\omega}_0$, is not, as the speed of m_0 is u - an extrinsic quantity and $\boldsymbol{\omega}_0$ has handed direction only when $u \neq 0$. Call $\boldsymbol{\omega}_0$ an out-looking intrinsic quantity or external proper quantity. In other words, $\boldsymbol{\omega}_0$ is an out-looking shell, while $\boldsymbol{\omega}_{\mathrm{v}}^{(1)}$, $\boldsymbol{\omega}_0^{(1)}$ and $\left(\omega_0^{(1)}\right)_{\mathrm{ft}} = \sum_{i=2}^{\infty} \omega_{\mathrm{f}}^{(i)}$ make up an intrinsic core as a whole - intrinsic construction of zeronic field. A particle composed of monoset of zerons is called a monocore particle, and it is called a bicore particle if it is composed of biset of zerons. All other relevant quantities and all other particles can be classified on the analogy of these.

At the state with the minimum potential $V_{\min} = 0$, a stable intrinsic core, for short, a sic − a sic in the intrinsic 4-dimensional space-time can be formed in zeronic field. Let the potential energy

$$\phi = -\int f\,\mathrm{d}r = -\int \frac{\hbar c}{r^2}\mathrm{d}r = \frac{\hbar c}{r} - C \equiv 0, \tag{85}$$

where

$$f = mc|\omega| = \frac{\hbar|\omega|^2}{c} = \frac{\hbar c}{r^2} \quad \left(c = |\omega| r\right) \tag{86}$$

then

$$C = \frac{\hbar c}{r_C} = \begin{cases} 0 & \text{when } r_C = \infty \text{ and } f = 0 \\ \text{nonzero constant} & \text{when } r_C < \infty \text{ and } f > 0. \end{cases} \tag{87}$$

This is the inevitable outcome resulting from the relativistic relation between the whole-space-distribution and the finite-space-appearance of a nonzero-proper-mass particle. When $f > 0$, if $r < r_C$, then $\phi > 0$, which means the centrifugal tendency is more than the centripetal tendency and hence $r \to r_C$; if $r > r_C$, then $\phi < 0$, which means the centrifugal tendency is less than the centripetal tendency and hence $r \to r_C$, too. So there is a stable state in which the centrifugal tendency is counteracted by a constant centripetal tendency at where $r_C < \infty$. This state is a stable bound state, while another stable state existing at where $r_C = \infty$ is a stable free state. The forming of such a stable bound state is a necessary, but not sufficient, condition to form a ground state of a nonzero-proper-mass particle. Another relevant necessary condition is that the handed direction of $\boldsymbol{\omega}_0^{(i+1)}$ must be different from that of $\boldsymbol{\omega}_0^{(i+2)}$ and that of $\boldsymbol{\omega}_\mathrm{v}^{(i+2)}$. If observing $\boldsymbol{\omega}_0^{(i+1)}$ at the level $i+2$, only a relative waving velocity $(\boldsymbol{\omega}_0^{(i+1)} - \boldsymbol{\omega}_0^{(i+2)})$, or $(\boldsymbol{\omega}_0^{(i+1)} - \boldsymbol{\omega}_\mathrm{v}^{(i+2)})$, can be measured. Then the attractive force maintaining the waving velocity is proportional to the square of this vectorial difference. To get the greatest attractive force and the stable structure, the handed direction of $\boldsymbol{\omega}_0^{(i+1)}$ must be different from that of $\boldsymbol{\omega}_0^{(i+2)}$ and that of $\boldsymbol{\omega}_\mathrm{v}^{(i+2)}$, and then $\boldsymbol{\omega}_0^{(i+1)}$ and $\boldsymbol{\omega}_\mathrm{v}^{(i+1)}$ have the same handed direction.

If write

$$Z^{i\cdots k} = \sum_{n=i}^{k} Z^{(n)} \qquad Z^{i\cdots} = Z^{i\cdots\infty} \qquad Z^{\cdots k} = Z^{0\cdots k} \tag{88}$$

and

$$\left(Z^{i\cdots k}\right)^2 = \left(\sum_{n=i}^{k} Z^{(n)}\right)^2 \qquad \left(Z^{i\cdots}\right)^2 = \left(Z^{i\cdots\infty}\right)^2 \qquad \left(Z^{\cdots k}\right)^2 = \left(Z^{0\cdots k}\right)^2 \tag{89}$$

where Z represents a scalar, then, when the handed directions of $\boldsymbol{\omega}_0^{(i+1)}$ and $\boldsymbol{\omega}_0^{(i+2)}$ are different,

$$\left(\omega_0^{(i)}\right)_{\mathrm{ft}} = \omega_\mathrm{f}^{(i+1)\cdots} = \pm\sum_{l=i+1}^{\infty} (-1)^l \left|\omega_\mathrm{f}^{(l)}\right| \tag{90}$$

and

$$\left(\omega_0^{(i)}\right)_{\text{ft}}^2 = \left(\omega_{\text{f}}^{(i+1)\cdots}\right)^2 = \left(\sqrt{2}-1\right)^2 \left(\sum_{l=i+1}^{\infty} (-1)^l \frac{|\omega_0|}{\left(\sqrt{2}\right)^l}\right)^2 = \left(3-2\sqrt{2}\right)^2 \left(\frac{1}{2}\right)^i \omega_0^2 \tag{91}$$

$$= \left(3-2\sqrt{2}\right)^2 \left(\omega_0^{(i)}\right)^2 .$$

If what is viewed at the observing level $i=1$ is a complete "stable intrinsic core" in ground state with interference when taking part in probability amplitude [Ref. 3: p 1-10, expressions (1.7)] ,

$$\left(m_{\text{sic}}\right)^2 = \left(m_{\text{v}}^{(1)} + m_0^{(1)} + \left(m_0^{(1)}\right)_{\text{ft}}\right)^2 = \left(Sm_0^{(1)}\right)^2 \tag{92}$$

and

$$\left(\omega_{\text{sic}}\right)^2 = \left(\left|\omega_{\text{v}}^{(1)}\right| + \left|\omega_0^{(1)}\right| + \left|\left(\omega_0^{(1)}\right)_{\text{ft}}\right|\right)^2 = \left(S\omega_0^{(1)}\right)^2 \tag{93}$$

where

$$S = 5-2\sqrt{2} \tag{94}$$

is important interaction parameter in zeronic field.

4 Total curvature and electron mass

The total curvature is, if it is a positive definite quantity, [Ref. 5: Chap. 7, Sect. 16, expressions (6) and (7)]

$$|K| = \bar{D}/D = \lim \left(\Delta\bar{\sigma}/\Delta\sigma\right) \tag{95}$$

where $D=\sqrt{EG-F^2}$ indicates the first fundamental quantities in the first fundamental form of the surface, $\Delta\sigma$ is an element of the surface and those with a bar each belongs to the spherical image of the unit sphere. If the surface here is also a sphere concentric with the unit sphere, then

$$|K| = \bar{r}^2/r^2 = 1/r^2 \tag{96}$$

which is right describing the probability in bound state if $\boldsymbol{r} = \boldsymbol{r}^{(i)} = \boldsymbol{r}_2^{(i)} - \boldsymbol{r}_1^{(i)}$ meeting the expression (81) in which the bound state at $r = r'' < \infty$ and the free state at $r = r' = \infty$ are twins in intrinsic space. With this understanding, the probability amplitude for a particle ordered i to go from $\boldsymbol{r}_1^{(i)}$ to $\boldsymbol{r}_2^{(i)}$ is given as $e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}^{(i)}/\hbar}/r^{(i)}$ [Ref. 3: p. 3-4, expression (3.7) with p. 1-10, expression (1.6)] and the relevant probability is

$$P^{(i)} = \left|\frac{e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}^{(i)}/\hbar}}{\boldsymbol{r}^{(i)}}\right|^2 = \frac{1}{\left(r^{(i)}\right)^2} = \left(\frac{c}{\hbar}\right)^2 \left(m^{(i)}\right)^2 , \tag{97}$$

where $m^{(i)}$ is the corresponding mass meeting

$$m^{(i)} r^{(i)} = \frac{\hbar}{c} \quad \left(\left|\omega^{(i)}\right| r^{(i)} = c\right). \tag{98}$$

For a sic,

$$m_{\text{sic}} = m_{\text{v}}^{(1)} + m_0^{(1)} + \left(m_0^{(1)}\right)_{\text{ft}} = \left(5-2\sqrt{2}\right) m_0^{(1)} = \left(5-2\sqrt{2}\right)\frac{\hbar}{c}\frac{1}{r_0^{(1)}} \tag{99}$$

and

$$\frac{1}{r_{\text{sic}}} = m_{\text{sic}} c/\hbar = \left(5-2\sqrt{2}\right)\frac{1}{r_0^{(1)}}. \tag{100}$$

The intrinsic force to maintain the sic in it is

$$f_{\text{in sic}} = m_{\text{sic}} c^2 \big/ r_{\text{sic}} = \left(5-2\sqrt{2}\right)^2 \hbar c \frac{1}{\left(r_0^{(1)}\right)^2} = \left(5-2\sqrt{2}\right)^2 \hbar c \sum_{i=1}^{\infty} \frac{1}{\left(r_{\text{v}}^{(i+1)}\right)^2} \tag{101}$$

where $\frac{1}{\left(r_{\text{v}}^{(i+1)}\right)^2} = P_{\text{v}}^{(i+1)}$. Let a dimensionless normalizing variable $1/a$ be multiplied to $1/r_0^{(1)}$, to get

$$f_{\text{in sic}} = \left(5-2\sqrt{2}\right)^2 \hbar c \frac{1}{\left(a r_0^{(1)}\right)^2} = 100\% = 1. \tag{102}$$

This right means $\left(\frac{1}{a}\right)^2$ is normalizing factor and that on the unit sphere let the r^2 in $|K|$ become

$$r^2 \to \overline{r}^2 = 1 \tag{103}$$

or

$$r^2 \big/ \overline{r}^2 \to 1, \tag{104}$$

this is relatively normalized unit in mathematics equivalent to normalization in physics. Furthermore, take

$$a r_0^{(1)} = \rho_0^{(1)}. \tag{105}$$

Since

$$\rho_0^{(1)} \in \left\{ r_0^{(1)} \middle| \left(r_0^{(1)}\right)^{-2} = \sum_{i=1}^{\infty} \left(r_{\text{v}}^{(i+1)}\right)^{-2} = \sum_{i=1}^{\infty} P_{\text{v}}^{(i+1)} \right\}, \tag{106}$$

the $f_{\text{in sic}}$ can be still written as

$$f_{\text{in sic}} = \left(5-2\sqrt{2}\right)^2 \hbar c \frac{1}{\left(r_0^{(1)}\right)^2} = \left(5-2\sqrt{2}\right)^2 \frac{c^3}{\hbar}\left(m_0^{(1)}\right)^2 = 1. \tag{107}$$

From this expression,

$$\left(m_0^{(1)}\right)_{\text{in sic}} = \sqrt{\frac{\hbar}{c^3}} \Big/ \left(5-2\sqrt{2}\right) = 9.11 \times 10^{-31}\,(\text{kg}) \tag{108}$$

which is right the proper mass of an electron - m_e - a stable particle in sic, consisting of an infinite set of zerons distributed in whole space-time, and taking the ∞-ordered zeron as its representative revolving within a relativistically contracted local region.

To see more about the important role of parameter $S = 5-2\sqrt{2}$ in sic (stable intrinsic core), the calculating consequence of one prepared paper titled “Calculating of heavier baryon masses in octet and decuplet with essential quantization plus graph theory” is introduced as follows.

$$
\begin{aligned}
&m_N + \left(1 + 1\cdot\frac{s}{2}\right)m_\pi = 1231\,\text{MeV/c}^2 \Rightarrow m_\Delta \\
&m_N + \left(1 + 2\cdot\frac{s}{2}\right)m_\pi = 1382\,\text{MeV/c}^2 \Rightarrow m_{\Sigma*} \\
&m_N + \left(1 + 3\cdot\frac{s}{2}\right)m_\pi = 1534\,\text{MeV/c}^2 \Rightarrow m_{\Xi*} \\
&m_N + \left(1 + \frac{1}{4}\cdot\frac{s}{2}\right)m_\pi = 1116\,\text{MeV/c}^2 \Rightarrow m_\Lambda \\
&m_N + \left(1 + \frac{3}{4}\cdot\frac{s}{2}\right)m_\pi = 1193\,\text{MeV/c}^2 \Rightarrow m_\Sigma \\
&m_N + \left(2 + \frac{2}{3}\cdot\frac{s}{2}\right)m_\pi = 1320\,\text{MeV/c}^2 \Rightarrow m_\Xi \\
&m_N + \left(2 + 3\cdot\frac{s}{2}\right)m_\pi = 1672\,\text{MeV/c}^2 \Rightarrow m_\Omega
\end{aligned}
\tag{109}
$$

where N is nucleon p or n , π is pion π^+, π^- or $(\pi^+ + \pi^-)/2$.

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（For reviewer – not published）

Contents

Important paragraphs or lines are marked with dark blue color.

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About sic (stable intrinsic core)

1) $\omega_0^{(1)}$

As known,

$$\left(\omega_0^{(1)}\right)^2 = \left(\omega_v^{(2)}\right)^2 + \left(\omega_0^{(2)}\right)^2 \Rightarrow \boldsymbol{\omega}_0^{(1)} = \boldsymbol{\omega}_v^{(2)} + \boldsymbol{\omega}_0^{(2)}$$

$$\left(\omega_0^{(2)}\right)^2 = \left(\omega_v^{(3)}\right)^2 + \left(\omega_0^{(3)}\right)^2 \Rightarrow \boldsymbol{\omega}_0^{(2)} = \boldsymbol{\omega}_v^{(3)} + \boldsymbol{\omega}_0^{(3)}$$

.....

$$\boldsymbol{\omega}_0^{(1)} = \sum_{n=2}^{\infty} \boldsymbol{\omega}_v^{(n)}.$$

Quoting expression (45)

$$\left(\omega_0^{(i)}\right)^2 = \left(\omega_v^{(i)}\right)^2 = \frac{\left(\omega_0^{(i-1)}\right)^2}{2} \quad (i = 1, 2, 3, \cdots),$$

so,

$$\left(\omega_0^{(1)}\right)^2 = \left(\omega_v^{(2)}\right)^2 + \left(\omega_0^{(2)}\right)^2 = \left(\omega_v^{(2)}\right)^2 + \left(\omega_v^{(3)}\right)^2 + \left(\omega_0^{(3)}\right)^2 = \cdots$$
$$= \sum_{i=2}^{\infty} \left(\omega_v^{(i)}\right)^2 = \sum_{i=2}^{\infty} \left(\omega_0^{(i)}\right)^2$$

that is

$$\left(\omega_0^{(1)}\right)^2 = \sum_{i=2}^{\infty} \left(\omega_0^{(i)}\right)^2$$

and

$$\left(m_0^{(1)}\right)^2 = \sum_{i=2}^{\infty} \left(m_0^{(i)}\right)^2 .$$

2) $\omega_v^{(1)}$

Again quoting expression (45),

$$\left(\omega_v^{(1)}\right)^2 = \left(\omega_0^{(1)}\right)^2 = \sum_{i=2}^{\infty} \left(\omega_v^{(i)}\right)^2$$

and

$$\left(m_{\mathrm{v}}^{(1)}\right)^2=\sum_{i=2}^{\infty}\left(m_{\mathrm{v}}^{(i)}\right)^2 .$$

3) $\left(m_0^{(1)}\right)_{\mathrm{ft}}$

Expression (92) means, when $\omega_0^{(i)}$ and $\omega_{\mathrm{f}}^{(i+1)}$ change signs alternately,

$$\left(\omega_0^{(i)}\right)_{\mathrm{ft}}^2=\left(\omega_{\mathrm{f}}^{(i+1)\cdots}\right)^2=\left(\sum_{l=i+1}^{\infty}\omega_{\mathrm{f}}^l\right)^2=\left[\pm\sum_{l=i+1}^{\infty}(-)^l\left|\omega_{\mathrm{f}}^l\right|\right]^2$$

$$=\left[\pm\sum_{l=i+1}^{\infty}(-)^l\left(\left|\omega_0^{(l-1)}\right|-\left|\omega_0^{(l)}\right|\right)\right]^2=\left(\sqrt{2}-1\right)^2\left[\sum_{l=i+1}^{\infty}(-)^l\frac{\left|\omega_0\right|}{\left(\sqrt{2}\right)^l}\right]^2$$

$$=\left(\sqrt{2}-1\right)^2\left[(-)^{i+1}\frac{\left|\omega_0\right|}{\left(\sqrt{2}\right)^{i+1}}+(-)^{i+2}\frac{\left|\omega_0\right|}{\left(\sqrt{2}\right)^{i+2}}+(-)^{i+3}\frac{\left|\omega_0\right|}{\left(\sqrt{2}\right)^{i+3}}+\cdots\right]^2$$

$$=\left(\sqrt{2}-1\right)^2\left\{(-)^{i+1}\left(\sqrt{2}-1\right)\left|\omega_0\right|\left[\frac{1}{\left(\sqrt{2}\right)^{i+2}}+\frac{1}{\left(\sqrt{2}\right)^{i+4}}+\frac{1}{\left(\sqrt{2}\right)^{i+6}}+\cdots\right]\right\}^2$$

$$=\left(\sqrt{2}-1\right)^2\left\{\left(\sqrt{2}-1\right)^2\left|\omega_0\right|^2\left[\frac{1}{\left(\sqrt{2}\right)^i}\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k\right]^2\right\}$$

$$=\left(3-2\sqrt{2}\right)^2\left(\frac{\omega_0}{\left(\sqrt{2}\right)^i}\right)^2=\left(3-2\sqrt{2}\right)^2\left(\omega_0^{(i)}\right)^2 ,$$

that is

$$\left(\omega_0^{(1)}\right)_{\mathrm{ft}}^2=\left(3-2\sqrt{2}\right)^2\left(\omega_0^{(1)}\right)^2$$

and

$$\left(m_0^{(1)}\right)_{\mathrm{ft}}^2=\left(3-2\sqrt{2}\right)^2\left(m_0^{(1)}\right)^2 .$$

Therefore

$$m_{\mathrm{v}}^{(1)}+m_0^{(1)}+\left(m_0^{(1)}\right)_{\mathrm{ft}}=\left[1+1+(3-2\sqrt{2})\right]m_0^{(1)}=(5-2\sqrt{2})\ m_0^{(1)}$$

or

$$\left|\omega_{\mathrm{v}}^{(1)}\right|+\left|\omega_0^{(1)}\right|+\left|\left(\omega_0^{(1)}\right)_{\mathrm{ft}}\right|=(5-2\sqrt{2})\left|\omega_0^{(1)}\right| .$$